TABLE 2.—*Data obtained by weighing method.*

[Surface area, 75.18 cm.². Volume, 44.36 cm.³.]

Date.	Hour.	Temp.	Pressure.	Weight of ice.	Loss in weight per hour.	Remarks.
1906.		°*C.*	*Inches.*	*Grams.*	*Grams.*	
March 4......	9	−10.5	30.30	40.810	....	Fair.
Do	10	−10.1	30.30	40.746	0.064	
Do	11	− 9.6	30.31	40.671	0.075	
Do	12	− 8.5	30.31	40.606	0.065	
Do	1	− 8.0	30.30	40.539	0.067	
Do	2	− 7.3	30.28	40.473	0.066	
Do	3	− 5.1	30.29	40.406	0.067	
Do	4	− 5.2	30.30	40.341	0.065	
Do	5	− 5.4	30.30	40.273	0.068	
Average ...	9 to 5	− 7.7†	30.30†	....	0.067	
March 4–5. Average ...	5 to 9	....	(30.29)	....	0.060	Minimum temperature −12° C.
March 5......	9	− 8.9	30.28	39.313	....	Fair.
Do	10	− 8.2	30.26	39.251	0.062	
Do	11	− 7.3	30.25	39.186	0.065	
Do	12	− 6.1	30.25	39.116	0.070	
Do	1	− 5.0	30.25	39.054	0.062	
Do	2	− 4.9	30.26	38.994	0.060	
Do	3	− 5.2	30.27	38.931	0.063	
Do	4	− 6.0	30.27	38.863	0.068	
Do	5	− 7.8	30.27	38.799	0.064	
Average ...	9 to 5	− 6.4†	30.26†	....	0.064	
March 5–6. Average ...	5 to 9	....	(30.18)	....	0.061	Minimum temperature −10.5° C.
March 6......	9	− 6.0	30.10	37.823	....	Cloudy in a. m.
Do	10	− 5.4	30.11	37.757	0.066	Fair in p. m. with heavy wind.
Do	11	− 4.3	30.08	37.690	0.067	
Do	12	− 3.6	30.08	37.620	0.070	
Do	1	− 3.4	30.07	37.555	0.065	
Do	2	− 2.2	30.09	37.487	0.068	
Do	3	− 2.0	30.09	37.418	0.069	
Do	4	− 1.5	30.09	37.346	0.072	
Do	5	− 1.5	30.08	37.271	0.075	
Average ...	9 to 5	− 3.3†	30.09†	....	0.069	

† The first and last readings of the series were given half weight.

[*To be continued.*]

HARMONIC ANALYSIS OF THE DIURNAL BAROMETRIC CURVE AT WASHINGTON, D. C.

By W. J. BENNETT, B. S., Observer. Dated Charlotte, N. C., November 12, 1906.

Averages of hourly barometric readings at Washington, D. C., for fourteen years, 1891–1904, show a diurnal variation of .0637 inch, with a maximum occurring about 10 a. m., and a minimum about 4 p. m. There is another maximum occurring about 11 p. m., and another minimum at 3 a. m., tho the difference between these is only .0073 inch. Similar phenomena are found at all stations when hourly barometer observations are averaged, but at places having an oceanic climate the two maxima and the two minima are nearly equal. The day maximum and minimum might be accounted for by the diurnal change in temperature, the maximum barometer being connected with the low morning temperature, and the minimum barometer with the high afternoon temperature, but no such simple explanation will account for the night maximum and minimum. The method of harmonic analysis has been employed by Prof. F. N. Cole,[1] and by others to throw some light on the subject. When diurnal barometer curves from various stations are analyzed into their components it is found that the first, or diurnal component, varies greatly from place to place, its phases differing by several hours, and its amplitude, very large over continental interiors, becoming very small over the ocean. This component, then, appears to be due largely to local causes, especially to the diurnal range in temperature. The second component, with a semidiurnal period, is quite uniform over the world, and does not seem affected to any great degree by local causes. The third and fourth components have much smaller amplitudes than the first and second. They are more uniform in phase and amplitude than the first, but less uniform than the second.

[1] The Diurnal Variation of Barometric Pressure. By Frank N. Cole. Weather Bureau Bulletin No. 6. Washington, 1892.

The object of my study was to compare data obtained from the analyses of barometer curves for several different periods of years at the same place, and by such comparison to obtain some idea as to the reality, magnitude, and fluctuations of the first four components. Averages were taken of hourly barometer readings at Washington, D. C., (corrected for temperature and instrumental error only) for the periods 1891–1894, 1895–1899, and 1900–1904, and independent calculations were made for the whole 14-year period 1891–1904.

An outline of the mathematical development of the equations used may render clearer the results obtained. The ordinary cosine curve has the equation

$$y=P\cos x,$$

where P is the amplitude, or one-half the difference between the maximum and minimum ordinates. If the first maximum does not fall on the line $x=0$, the equation becomes

$$y=P\cos(x-M),$$

where M is the epoch or distance of the first maximum from the line $x=0$. For a curve of twice the frequency the equation becomes

$$y=P_2\cos 2(x-M_2),$$

and for a curve of n times the frequency,

$$y=P_n\cos n(x-M_n).$$

For a curve made up of a number of superimposed cosine curves, as the diurnal barometer curve is supposed to be, we have

$$Y=y_1+y_2+y_3 \ldots\ldots +y_n,$$

and

$$Y=P_1\cos(x-M_1)+P_2\cos 2(x-M_2)\ldots\ .\ +P_n\cos n(x-M_n).$$

Expanding:

$$Y=P_1\cos x\cos M_1+P_1\sin x\sin M_1+P_2\cos 2x\cos 2M_2+P_2\sin 2x\sin 2M_2+\ldots\ldots+P_n\cos nx\cos nM_n+P_n\sin nx\sin nM_n.$$

Taking only four components, and letting

$Q_1=P_1\cos M_1$; $Q_n=P_n\cos nM_n$; and $R_1=P_1\sin M_1$; $R_n=P_n\sin nM_n$, we have:

$$Y=Q_1\cos x+R_1\sin x+Q_2\cos 2x+R_2\sin 2x+Q_3\cos 3x+R_3\sin 3x+Q_4\cos 4x+R_4\sin 4x.$$

There are eight unknown quantities, and if we have hourly means there will be twenty-four equations as x varies from 15° to 360°. The first and last of these will be:

$$Y_1=Q_1\cos 15^\circ+R_1\sin 15^\circ+Q_2\cos 30^\circ+R_2\sin 30^\circ+Q_3\cos 45^\circ+R_3\sin 45^\circ+Q_4\cos 60^\circ+R_4\sin 60^\circ.$$

$$Y_{24}=Q_1\cos 360^\circ+R_1\sin 360^\circ+Q_2\cos 720^\circ+R_2\sin 720^\circ+Q_3\cos 1080^\circ+R_3\sin 1080^\circ+Q_4\cos 1440^\circ+R_4\sin 1440^\circ.$$

By the principle of least squares, these equations may be reduced to eight, of which the first will be

$$12\,Q_1=Y_{24}-Y_{12}+(Y_1-Y_{13}+Y_{23}-Y_{11})\cos 15^\circ+(Y_2-Y_{14}+Y_{22}-Y_{10})\cos 30^\circ+(Y_3-Y_{15}+Y_{21}-Y_9)\cos 45^\circ+(Y_4-Y_{16}+Y_{20}-Y_8)\cos 60^\circ+(Y_5-Y_{17}+Y_{19}-Y_7)\cos 75^\circ.$$

The equations are given in full by Professor Ferrel (Report *of the Chief Signal Officer, 1885, Part 2, page 342*).

From the values of Q's and R's so obtained the values of P's and M's are calculated by the relations

$$\frac{R_n}{Q_n}=\frac{P_n\sin nM_n}{P_n\cos nM_n}=\tan nM_n$$

$$\log.\tan nM_n=\log.R_n-\log.Q_n.$$

$$P_n=\frac{Q_n}{\cos nM_n};\ \log.P_n=\log.Q_n-\log.\cos nM_n.$$

In these calculations careful attention must be paid to plus and minus signs, since $+Q_n$ and $+R_n$ put the angle nM_n in the first quadrant, $-Q_n$ and $+R_n$ in the second, $-Q_n$ and $-R_n$ in the third, and $+Q_n$ and $-R_n$ in the fourth. The factors P

from their nature are positive, since they are fractions of an inch barometric pressure, the length of the maximum ordinate for the given component.

TABLE 1.

Period.		P_1.	M_1.	P_2.	M_2.	P_3.	M_3.	P_4.	M_4.
		Inch.	*h. m.*	*Inch.*	*h. m.*	*Inch.*	*h. m.*	*Inch.*	*h. m.*
January,	1891-1894	.0123	6:50	.0189	9:27	.0080	2:21	.0037	3:53
	1895-1899	.0145	8:38	.0168	9:34	.0082	2:18	.0040	4:01
	1900-1904	.0199	5:59	.0201	9:28	.0089	2:18	.0044	4:03
	1891-1904	.0143	7:32	.0180	9:31	.0085	2:25	.0042	3:55
April,	1891-1894	.0211	7:01	.0185	10:04	.0011	7:08	.0016	6:17
	1895-1899	.0238	6:33	.0178	10:04	.0007	6:18	.0007	7:23
	1900-1904	.0244	6:24	.0180	9:59	.0011	5:55	.0012	8:25
	1891-1904	.0239	6:32	.0183	10:01	.0010	6:32	.0008	6:59
July,	1891-1894	.0219	7:58	.0147	10:39	.0014	6:55	.0014	6:21
	1895-1899	.0210	7:59	.0140	10:20	.0014	6:24	.0009	6:20
	1900-1904	.0210	7:16	.0155	10:23	.0026	6:21	.0008	6:07
	1891-1904	.0213	7:45	.0141	10:19	.0016	6:37	.0008	5:40
October,	1891-1894	.0180	6:56	.0185	9:47	.0025	2:05	.0003	5:27
	1895-1899	.0230	6:45	.0174	9:46	.0042	2:13	.0004	3:10
	1900-1904	.0164	7:02	.0213	9:44	.0042	2:00	.0011	4:27
	1891-1904	.0195	6:44	.0192	9:46	.0031	2:08	.0004	4:23
Year,	1891-1894	.0164	7:01	.0168	9:53	.0013	2:27	.0009	4:52
	1895-1899	.0203	7:11	.0168	9:56	.0021	2:15	.0005	4:38
	1900-1904	.0208	6:57	.0176	9:54	.0030	2:11	.0007	4:01
	1891-1904	.0192	7:02	.0174	9:55	.0018	2:18	.0007	4:40

TABLE 2.

Month.	P_1.	M_1.	P_2.	M_2.	P_3.	M_3.	P_4.	M_4.
	Inch.	*h. m.*	*Inch.*	*h. m.*	*Inch.*	*h. m.*	*Inch.*	*h. m.*
January	.0143	7:32	.0180	9:31	.0085	2:25	.0042	3:55
February	.0179	6:34	.0188	9:42	.0066	2:36	.0009	5:20
March	.0190	6:33	.0186	9:41	.0032	2:03	.0015	6:37
April	.0239	6:32	.0183	10:01	.0010	6:32	.0008	6:59
May	.0241	6:58	.0173	10:08	.0026	6:10	.0013	6:14
June	.0214	7:38	.0142	10:22	.0023	6:34	.0008	7:09
July	.0213	7:45	.0141	10:19	.0016	6:37	.0008	5:40
August	.0185	8:08	.0152	10:34	.0009	5:29	.0005	6:10
September	.0218	7:35	.0177	10:09	.0010	2:53	.0007	4:30
October	.0195	6:44	.0192	9:46	.0031	2:08	.0004	4:23
November	.0176	6:19	.0188	9:23	.0050	2:07	.0014	4:27
December	.0132	6:36	.0197	9:30	.0043	1:40	.0031	4:06
Year	.0192	7:02	.0174	9:55	.0018	2:18	.0007	4:40

In Table 1 are given values of *P's* and *M's* obtained from independent calculations for each of the four periods, 1891–1894, 1895–1899, 1900–1904, and 1891–1904. The remarkably close agreement among the figures would seem to indicate that the four components are not mathematical fictions, but physical realities. Table 2 gives monthly and annual values for the whole 14-year period, 1891–1904.

It will be seen that the amplitude of the first component is greatest in early summer and least in early winter, the difference between the values for May and December being .0109 inch. This variation agrees closely with the mean daily range in temperature, which is greatest in May and least in January. Following are the mean monthly temperature ranges for the same fourteen years:

Month.	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Mean in °F.	15.1	16.0	17.4	19.3	21.2	19.3	19.0	18.6	19.1	19.0	17.8	16.7

The time of the maximum of the first component averages about an hour later than that of the lowest temperature for the day, but considering it by months we notice a double oscillation, as it occurs later in summer and winter than in spring and autumn. This may possibly be due to the fact that the lowest temperature occurs earlier and the highest later in summer than in winter. In the spring the hour of the lowest temperature recedes more rapidly than that of the highest advances, the tendency being to throw the epoch of the first component earlier. As summer advances the highest temperature occurs later, while the lowest changes its position but slightly, and the epoch is thrown later. During the autumn the highest temperature comes earlier, while the change in the lowest is still small, throwing the epoch earlier again. During the winter the lowest temperature advances while the highest remains nearly stationary, and thus the cycle is completed by the epoch occurring later. It seems, then, that the first component is largely the effect of local temperature changes, and this theory is made more probable by the fact that at places having an oceanic climate, with small diurnal temperature range, the amplitude of the first component is the least, while at places having a continental climate, with great diurnal temperature range, the amplitude is greatest.

It must be noted here that since the temperature change from lowest to highest is nearly twice as rapid as that from highest to lowest, its effect upon the barometer can not be ascertained accurately by a method of harmonic analysis which forces the maximum and minimum of the first component to occur exactly twelve hours apart. Some of the temperature effect must be taken up by the second component, which has maxima and minima occurring six hours apart, and which will have a maximum later and a minimum earlier than the maximum and minimum, respectively, of the first component. An attempt to find the value of the temperature effect thus lost by the first component and gained by the second, and to eliminate roughly from the diurnal curve the direct effect of temperature, was made in the following manner: As the lowest temperature occurs about 6 a. m. and the highest about 3 p. m., eight instead of fourteen values of Y were taken in the interval between 3 p. m. and 6 a. m., and analysis made on the basis of eighteen observations, with temperature extremes occurring at intervals of nine. The amplitude of the first component thus obtained was .0253 inch, or .0061 greater than that obtained before. The epoch fell at 6:16 a. m. Plotting this curve, and then subtracting its ordinates from the corresponding ordinates of the normal barometer curve, gave a curve from which the direct effect of temperature was eliminated, and this was then analyzed by the usual method. The values then obtained for the second, third, and fourth components were as follows:

P_2 .0123	P_3 .0008	P_4 .0017
M_2 10:11 a. m.	M_3 3:45 a. m.	M_4 3:21 a. m.

The second component has its greatest amplitude in winter and its least in summer, the variation being inversely as the amplitude of the first component, but only about half as great, or .0056. A cause for this and a second possible cause for the variation of the first component may be found in the fact that the interval between the lowest and highest temperatures for the day is greater in summer than in winter. More of the temperature effect would therefore appear in the first component in summer and less in winter. The reverse would be true in regard to the second component. The epoch of the second component also shows an annual variation, being about an hour later in summer than in winter. This variation is quite uniform over the world and seems independent of local conditions, but its cause is not understood. After eliminating roughly the direct temperature effect, as by the method above given, the amplitude of the second component appears reduced from .0174 to .0123, or by about a third. The smaller figure is still considerable.

If the semidiurnal component is due to a tidal effect of the sun, then there should be a lunar barometric tide also, and this of greater amplitude. In studying this question the barograph trace sheets at Charlotte, N. C., (those of Washington not being available) were examined, and twenty-four readings were taken every twenty-four hours and fifty minutes, beginning with the moon's culmination. Averages were

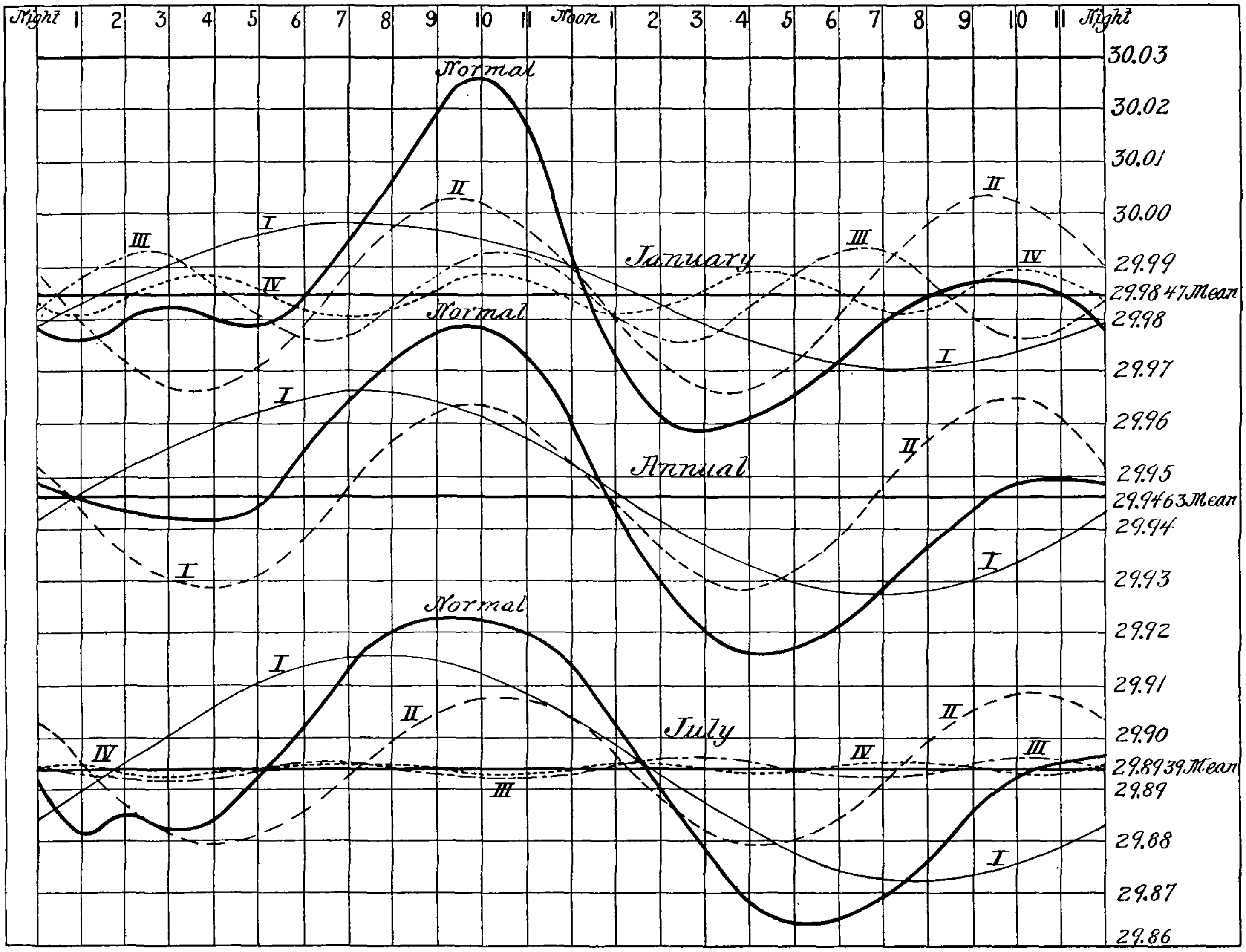

FIG. 1.—The diurnal barometric curve at Washington, D. C.

obtained for a year and the resulting curve analyzed. The amplitudes and epochs of the first four components were as follows, epochs being reckoned from the moon's culmination:

P_1 .00018	P_2 .00092	P_3 .00044	P_4 .00002
M_1 5 h. 35 m.	M_2 0 h. 33 m.	M_3 6 h. 11 m.	M_4 3 h. 58 m.

It will be noted that the second component is the only one that seems to have a real value, but its amplitude is very small. A solar atmospheric tide would have a still smaller amplitude, and while it probably exists, would exert a scarcely appreciable effect upon the barometer. A gravitational explanation of the second component being unsatisfactory, we are forced to consider this component as a thermal effect, brought about in a manner difficult to understand, possibly thru radiation from the upper strata of the atmosphere or thru a vertical movement of the air.

The amplitude of the third component shows a double oscillation, being greatest in winter and summer and least in spring and autumn. This is not real, however, but is due to the fact that this component changes its phase at the equinoxes, the summer maxima falling at the hour of the winter minima. There is really a single variation with a range of from .0085 in winter to .0023 in summer. The epoch occurs about 2 a. m. in winter and about 6 a. m. in summer. It seems to be connected in some way with the annual movement of the sun in latitude.

The fourth component also has its greatest amplitude in winter and its least in summer, the annual variation being from .0042 to .0004. Its phase changes rapidly from month to month, the epoch in winter being about 4 a. m. and in summer about 7 a. m., making a reversal of phase, so that the summer maxima fall at the time of the winter minima. At other stations this same peculiarity appears, and there is a much greater amplitude in winter than in summer. Altho it is exceedingly difficult to assign any physical cause for this component, its characteristics are such that we can not regard it as the result of purely accidental fluctuations.

Components of higher order than the fourth may be obtained by analysis of the diurnal curve, but they are of less amplitude and greater irregularity than the first four, and there is little to indicate that they have any physical significance. It is possible that further study of the first and second components may prove these to be the only ones that are real. The invention of some method that will take into account in calculation the nature, time, and effect of the causes of the first two may reduce the higher components to negligible quantities and show that their appearance may be due to the fact that our present methods of harmonic analysis can not well take into account causes operating thru periods incommensurable with twenty-four hours, nor allow for effects varying except as cosine curves.